Lec 13: 02/29/2012

Photoelectric Absorption:

So far, we have discussed interaction of high energy charged particles and their energy loss. We now turn to interaction of high energy photons with other particles. We start this by discussing photoelectric absorption.

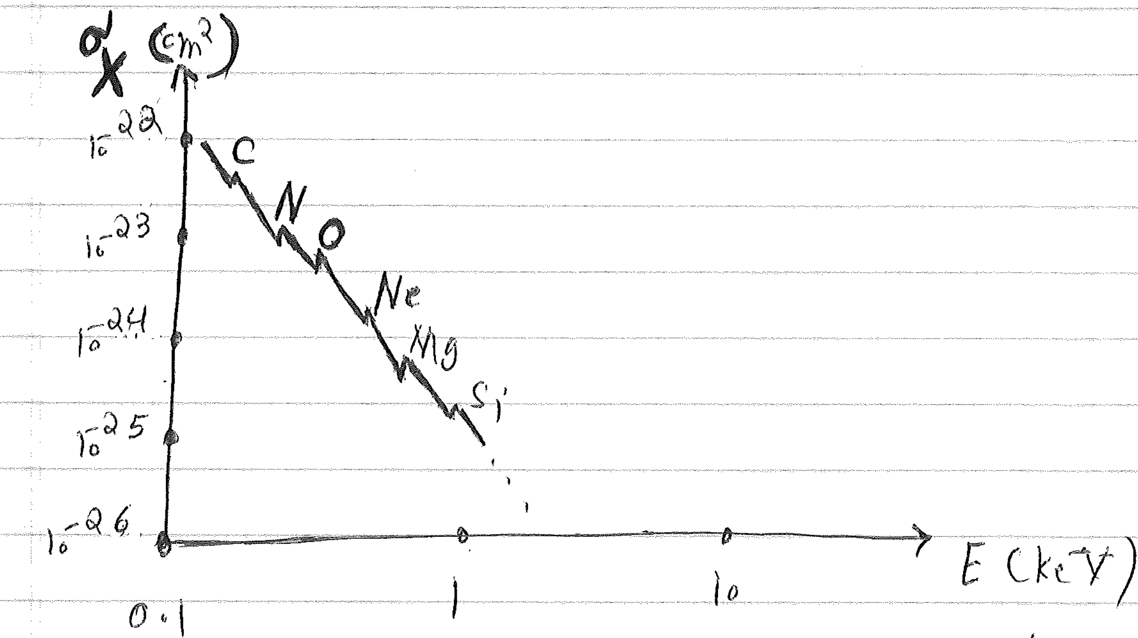
At low photon energies, $\hbar\omega \ll m_e c^2$, the dominant process by which photons lose energy is photoelectric absorption. Photoelectric, or bound-free, absorption is one of the principal sources of opacity in stellar interiors and stellar atmospheres. If the energy of the incident photon is $\hbar\omega$, it can eject electrons that have binding energies $E \leq \hbar\omega$. The energy levels within an atom for which $\hbar\omega = E$ are called "absorption edges". The cross-section for photoelectric absorption can be calculated by using the quantum theory of radiation. The cross-section for photons with energies $\hbar\omega \gg E$ (and $\hbar\omega \ll m_e c^2$) due to the ejection of electrons from the K-shells of atoms (i.e., the 1S level) is given by:

$$\sigma_K = \frac{64\pi e^{12} m_e^{3/2} Z^5}{3\sqrt{2}\,\hbar^4 c} \left(\frac{1}{\hbar\omega}\right)^{\frac{7}{2}}$$

Note the very strong dependence of σ_K on the atomic number Z. This implies that although heavy elements are very much less abundant than hydrogen, they make important contributions to the absorption cross-section, particularly at hard ultraviolet and X-ray energies.

An important application of photoelectric absorption is in the determination of the X-ray absorption coefficient for interstellar matter. To estimate this, it is necessary to add curves for different elements that have different cosmic abundances.

K-edges of heavier elements appear at higher energies, and provide the dominant source of opacity above those energies. The resulting total absorption cross-section is the sum of individual contributions by different elements, which is schematically shown in the following figure:



A useful linear interpolation formula for the X-ray optical depth is:

$$\tau_X(\hbar\omega) = 2\times 10^{-24}\left(\frac{\hbar\omega}{1\,\text{keV}}\right)^{-\frac{8}{3}} \int N_H\, dl$$

Here the column depth $\int N_H\, dl$ is expressed in particles per cm^2 and N_H is the number density of hydrogen atoms per cm^3. Because of the steep energy dependence of σ_X, photoelectric absorption is not generally important at energies $\hbar\omega \gtrsim 1\,keV$.

Compton Scattering:

In the Compton scattering process, the incoming high energy photons collide with stationary electrons and transfer some of their energy and momentum to the electrons. Consequently, the photons come out of the process with less energy and momentum.

Let us begin with a simplified classical treatment. An electron subject to the electromagnetic field of an incoming photon accelerates with a rate as $\frac{e}{m_e}E$, which results in a radiated power;

$$P = \frac{2}{3}\frac{e^4}{c^3 m_e^2}E^2$$

The incident flux is given by $f = c\,u_{rad}$, where u_{rad} is the radiation energy density: $u_{rad} = \frac{E^2}{4\pi}$. The scattering cross-section is defined as;

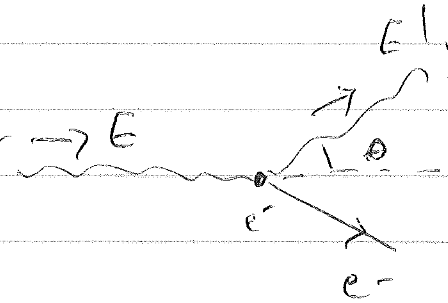
$$\sigma \equiv \frac{P}{f} = \frac{8\pi}{3} r_e^2 \qquad \left(r_e = \frac{e^2}{m_e c^2} \text{ : classical radius of the electron}\right)$$

This is the Thomson scattering cross-section:

$$\sigma_T = \frac{8\pi}{3} r_0^2 \simeq \frac{2}{3} \times 10^{-24}\ \mathrm{cm}^2$$

When the incoming photon is very energetic, the recoil of the electron cannot be ignored. The energy of the scattered photon is in general given by:

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)}$$



We see that $E' \simeq E$ for forward scattering ($\theta \simeq 0$), while $E' < E$ for all other angles. Note that $E' \simeq E$ regardless of the value of θ if $E \ll m_e c^2$.

The total cross section for Compton scattering using a full quantum is given by the Klein-Nishina formula:

$$\sigma_{K-N} = \pi r_e^2 \frac{1}{\varepsilon}\left[\left[1-\frac{2(\varepsilon+1)}{\varepsilon^2}\right]\ln(2\varepsilon+1)+\frac{1}{2}+\frac{4}{\varepsilon}-\frac{1}{2(2\varepsilon+1)^2}\right]$$

Here $\varepsilon = \frac{\hbar\omega}{m_e c^2}$. In the non-relativistic limit when $\hbar\omega \ll m_e c^2$ this is reduced to:

$$\sigma_{K-N} \simeq \frac{8\pi}{3} r_e^2 (1-2\varepsilon) \simeq \sigma_T \qquad \varepsilon \ll 1$$

In the relativistic limit when $\hbar\omega \gg m_e c^2$ we have:

$$\sigma_{K-N} = \pi r_e^2 \frac{1}{\varepsilon}\left(\ln 2\varepsilon + \frac{1}{2}\right)$$

The scattering cross-section can be used to calculate the power emitted by an energetic electron moving through photons. This is called the inverse Compton scattering, which is one of the most important processes in high-energy astrophysics. Scattering of low-energy photons by energetic electrons results in energy gain by the photons and energy loss for the electrons.

Let us consider the collision between a photon and a relativistic electron. If $\gamma \hbar\omega \ll m_e c^2$, then photon energy in the rest frame of the electron is $\hbar\omega' \ll m_e c^2$. ω' is related to ω through the Doppler shift formula:

$$\omega' = \gamma\omega\,(1+\beta\cos\theta)$$

Here θ is the angle of incidence in the lab frame. Compton scattering in the rest frame of the electron is simply Thomsen scattering (since $\hbar\omega' \ll m_e c^2$), and hence the radiated power is:

$$P' = \sigma_T\, c\, U'_{rad}$$

U'_{rad} is the energy density of radiation in the electron rest frame, which is related to U_{rad} according to:

$$U'_{rad} = [\gamma(1+\beta\cos\theta)]^2\, U_{rad}$$

Averaging over the angle of incidence θ, we find:

$$P' = \frac{4}{3}\sigma_T\, c\, U_{rad}\left(\gamma^2 - \frac{1}{4}\right)$$

We notice that power is Lorents invariant, which implies that in the lab frame we have,

$$P = \frac{4}{3} \sigma_T c U_{rad} \left(\gamma^2 - \frac{1}{4}\right)$$

The total Compton power, which is the difference between the radiated power and the incident power, is therefore given by:

$$P_{Comp} = \frac{4}{3} \sigma_T c U_{rad} (\gamma^2 - 1)$$

The appearance of γ indicates that Compton scattering is an effective process only for highly relativistic electrons. Thus, we are led to consider non-thermal particle populations. Even at $T \sim 10^9$ K, a typical electron has a velocity $v = \left(\frac{3kT}{m_e}\right)^{1/2}$ $\sim \frac{2}{3} c$, with a Lorentz factor $\gamma \simeq 1.4$. There are exceptions, however, like supermassive black holes at the galactic center whose X-ray flux appears to be due to the inverse Compton scattering by a very hot plasma ($T \gg 10^9$ K).

To underline the enormous impact of the Compton power on the emission of X-rays and γ-rays, consider the interaction of GeV cosmic ray electrons with the CMB photons in the interstellar medium. The CMB photons have an energy $E_0 \sim 10^{-3}$ eV. Upon scattering off GeV electrons with $\gamma \sim 2 \times 10^3$, photons with energy $E_{sc} \sim \gamma^2 E_0 \sim 4$ keV are produced. The scatterings therefore convert microwave photons into X-ray photons.

As mentioned earlier, the majority of Compton sources are non-thermal emitters. To describe them, we consider a power-law distribution:

$N(\gamma)\, d\gamma = K \gamma^{-q}\, d\gamma$

Here we use γ as the independent variable instead of energy. The total Compton power is:

$$P_{tot} = \int_{\gamma_1}^{\gamma_2} P_{Comp}(\gamma)\, N(\gamma)\, d\gamma$$

For a relativistic distribution $\gamma_1 = 1$ and $\gamma_2 = \gamma_{max}$. We then get:

$$P_{tot} = \frac{4}{3}\sigma_T\, c\, U_{rad}\, \frac{K}{3-x}\left(\gamma_{max}^{3-x} - 1\right) \simeq \frac{4}{3}\sigma_T\, c\, U_{rad}\, \frac{K}{3-x}\gamma_{max}^{3-x}$$

Spectral measurements of a Compton source can provide valuable clues about the physical state of the system, including the spectral index x and the high-energy cut off γ_{max}. A turnover in the spectrum can be an indication of significant cooling process, which delimits the efficiency of particle acceleration, or an evidence that the source has only been active for a short time.